Galaxy Evolution

One can imagine at least 5 types of galaxy evolution: photometric evolution (*i.e.,* changes in galactic luminosity, color, and spectra), chemical evolution, morphological evolution, dynamical evolution (*i.e.,* changes in the types of stellar orbits), and number evolution (*i.e.,* changes in the number of galaxies present in the universe). The last three are difficult to study analytically, since they are closely related. For example, collisions between galaxies may turn spirals into ellipticals, and consume smaller galaxies. At the same time, new galaxies may form from debris thrown out in a collision. In addition, morphological evolution of spirals can be caused by environmental effects: in a rich cluster, ram-pressure sweeping of the interstellar medium may occur as the galaxy passes through x-ray gas.

Fortunately, photometric and chemical evolution are more easily handled. Each can be parameterized in a fairly straightforward manner.

Photometric Evolution

[Tinsley 1980, *Fund. Cosmic Phys.,* **5,** 287]
[Rana 1991, *ARAA,* **29,** 129]
[Renzini & Buzzoni 1986, in *Spectral Evolution of Galaxies,* p. 195]
[Bruzual & Charlot 2003, *MNRAS,* **344,** 1000]
[Maraston 2005, *MNRAS,* **362,** 799]

The photometric evolution of galaxies is a particularly straightforward problem. There are several generally available computer codes that can trace the photometric evolution of a simple (or complex) stellar population throughout cosmic history. However to first order, it is possible to estimate such properties analytically.

In order to make the mathematics a bit more straightforward, let's choose a fiducial mass, m_1, which can be the mass of the Sun. Such a star will have a main-sequence luminosity, ℓ_1, and a main-sequence lifetime of τ_1. We will then define the variable, m as the dimensionless mass of a star, *i.e.,* for a star of mass M, $m = M/m_1$.

First we need is an Initial Mass Function (IMF) of stars, which gives the number of stars born as a function of mass. The IMF is usually written as a power law, *i.e.,*

$$\phi(m)dm = \phi_1 m^{-(1+x)} dm \tag{8.01}$$

where x describes the steepness of the function. [Most applications use the local neighborhood IMF slope of $x = 1.35$ found by Salpeter (1956), but more and more people are going to broken power laws, such as those of Miller & Scalo (1979), Kroupa *et al.* (1993), or Chabrier 2003).] The constant ϕ_1 is simply there for normalization purposes, so that a star cluster (or galaxy) with

total mass $\mathcal{M}_0$ satifies

$$\mathcal{M}_0 = \mathcal{M}_0 \int_0^\infty m\,\phi(m)\,dm \tag{8.02}$$

The second piece of information we need is the mass-luminosity relation for main-sequence stars. This is well known from stellar evolution, and, although the slope of this relation does change along the main sequence, for our purposes, we can approximate the function as a single power law, with slope α. Thus,

$$\ell_d = \ell_1 m^\alpha \tag{8.03}$$

where the subscript d represents the luminosities of dwarfs (*i.e.,* main sequence stars). While the mass-luminosity relation of the main sequence isn't quite a single power law, the approximation isn't too bad if you take $\alpha \sim 3.5$.

The third item needed is the length of time stars spend on the main sequence. The lifetime of a star is simply proportional to the energy available to the star divided by rate at which the star is emitting its energy, *i.e.,* its luminosity. Since the available energy is proportional to the available mass, and the luminosity is proportional to mass (though the mass-luminosity relation),

$$\tau \propto \frac{m}{\ell_d} \propto \left(\frac{m}{m^\alpha}\right) \propto m^{1-\alpha} \implies \tau = \tau_1 m^{1-\alpha} \tag{8.04}$$

Alternatively, this equation can be inverted. After a time t, the turnoff mass of a single-age stellar population will be

$$m_{tn} \propto t_{tn}^{1/(1-\alpha)} \implies m_{tn} = \left(\frac{t}{\tau_1}\right)^{1/(1-\alpha)} \tag{8.05}$$

The final pieces of information that are needed are the lifetime of a typical post-main sequence (giant) star (τ_g), the average luminosity of a giant star, ℓ_g, and the mass of a typical white dwarf, w. All of these can be derived (approximately) from models of stellar evolution. Note that $\tau_g \ll \tau_d$, so the stellar main-sequence lifetime is also the star's total lifetime.

We will now consider the evolution of a set of stars all born (with the same metallicity) at the same time. This is called a Simple Stellar Population (SSP). The properties of more complicated systems can be inferred from the summation of several SSPs.

Luminosity Evolution

Let's first calculate the luminosity evolution of a cluster (or galaxy) of stars with total mass $\mathcal{M}_0$ all born at the same time. The total luminosity of main sequence stars is easy to compute: all we have to do is sum up the luminosity of all stars on the main sequence. The lower end of the sum is the minimum mass of an energy-generating star; the upper end is defined by the main sequence turnoff. In other words,

$$\begin{aligned}
\mathcal{L}_D(t) &= \int_{m_L}^{m_{tn}} \mathcal{M}_0\, \phi(m)\, \ell_d(m)\, dm \\
&= \int_{m_L}^{m_{tn}} \mathcal{M}_0 \;\cdot\; \phi_1 m^{-(1+x)} \cdot \ell_1 m^{\alpha} dm \\
&= \mathcal{M}_0 \phi_1 \ell_1 \int_{m_L}^{m_{tn}} m^{\alpha-1-x} dm \\
&= \frac{\mathcal{M}_0 \phi_1 \ell_1}{\alpha - x} \left\{ m_{tn}^{\alpha-x} - m_L^{\alpha-x} \right\} \\
&= \frac{\mathcal{M}_0 \phi_1 \ell_1}{\alpha - x} \left\{ \left(\frac{t}{\tau_1}\right)^{\frac{\alpha-x}{1-\alpha}} - \left(\frac{t_L}{\tau_1}\right)^{\frac{\alpha-x}{1-\alpha}} \right\}
\end{aligned} \tag{8.06}$$

Note that since the exponent is negative, and low mass stars live (essentially) forever, the last term in the above equation is negligible. Thus, the total luminosity of dwarf stars is

$$\mathcal{L}_D(t) \approx \frac{\mathcal{M}_0 \phi_1 \ell_1}{\alpha - x} \left(\frac{t}{\tau_1}\right)^{(\alpha-x)/(1-\alpha)} \tag{8.07}$$

Calculating the total luminosity of giant stars is equally simple. First, note that the timescale for giant branch evolution is much faster than that for main sequence evolution. Thus, the key is to estimate the number of stars currently turning into giants; when this number is multiplied by the length of time a typical star remains a giant, and the mean luminosity of the star, the result is the total giant star luminosity. Now consider: the rate at which main sequence stars turn into giants is defined by how many stars are at the main sequence turnoff, and how much of the main sequence is eaten away per unit time interval. According to our definition of the initial mass function, the number of stars at the main-sequence turnoff is

$$\frac{dN(m)}{dm} = \phi(m) = M_0 \phi_1 m^{-(1+x)} \tag{8.08}$$

and the number of stars turning into red giants during a time Δt is

$$N_g = \frac{dN}{dt} \Delta t = \frac{dN}{dm} \cdot \frac{dm}{dt} \cdot \Delta t \tag{8.09}$$

so the total luminosity of giant stars is

$$\begin{aligned} \mathcal{L}_G &= \phi(m_{tn}) \cdot \frac{dm_{tn}}{dt} \cdot \tau_g \cdot \ell_g \\ &= \mathcal{M}_0 \phi_1 m_{tn}^{-(1+x)} \frac{dm_{tn}}{dt} \ell_g \, \tau_g \end{aligned} \tag{8.10}$$

If we now substitute for t for m using (8.05) take the derivative, and combine terms, the result is

$$\mathcal{L}_G = \frac{\mathcal{M}_0 \phi_1 \, \ell_g \, \tau_g}{\tau_1 (\alpha - 1)} \left(\frac{t}{\tau_1} \right)^{\frac{\alpha - x - 1}{1 - \alpha}} \tag{8.11}$$

Thus, the total luminosity of the stellar system, as a function of time, is

$$\mathcal{L}_T = \mathcal{L}_D + \mathcal{L}_G = \frac{\mathcal{M}_0 \phi_1 \ell_1}{\alpha - x} \left(\frac{t}{\tau_1} \right)^{\frac{\alpha - x}{1-\alpha}} + \frac{\mathcal{M}_0 \phi_1 \, \ell_g \, \tau_g}{\tau_1 (\alpha - 1)} \left(\frac{t}{\tau_1} \right)^{\frac{\alpha - x - 1}{1-\alpha}} \tag{8.12}$$

This can be simplified a bit if we define $G(t)$ as the ratio of giant star luminosity to dwarf star luminosity, *i.e.*,

$$G(t) = \frac{\mathcal{L}_G}{\mathcal{L}_D} = \frac{(\alpha - x) \, \ell_g \, \tau_g}{(\alpha - 1) \, \ell_1 \, \tau_1} \left(\frac{t}{\tau_1} \right)^{1/\alpha - 1} \tag{8.13}$$

Since $\alpha > 1$, the exponent of time in (8.13) is significantly less than one. Hence writing the expression for $G(t)$ in this way illustrates that the variable is only a weak function of time. This expression can then be further simplied by using (8.03) and (8.05)

$$\left(\frac{t}{\tau_1} \right)^{\frac{1}{\alpha - 1}} = \left(\frac{t}{\tau_1} \right)^{\frac{\alpha}{\alpha - 1}} \left(\frac{t}{\tau_1} \right)^{-\frac{\alpha - 1}{\alpha - 1}} = m^{-\alpha} \left(\frac{\tau_1}{t} \right) = \frac{\ell_1 \tau_1}{\ell_{tn} t} \tag{8.14}$$

Thus

$$G(t) = \frac{\alpha - x}{\alpha - 1} \left\{ \frac{\ell_g \, \tau_g}{\ell_{tn} \, t} \right\} \tag{8.15}$$

Note that the first part of this equation is very close to one. Furthermore, the term in brackets is simply the ratio of the total energy generated by the star on the giant branch to the total energy the star generated on the main sequence. Since stars turn off the main sequence when $\sim 10\%$ of their total fuel is gone, but eventually burn $\sim 70\%$ of their fuel, $G(t) \sim 6$. (From an observers point of view, it's a bit more complicated, since most of the light a giant star produces is red, while the main sequence

light may be blue. Thus, the exact value of $G(t)$ depends on the wavelength of observation: in the blue, $G(t) \sim 1$, but in the red, $G(t) > 10$.) Using this notation, the luminosity of a stellar population, as a function of time, is

$$\begin{aligned}\mathcal{L}_T(t) &= \mathcal{L}_D(t)\,\{1+G(t)\} \\ &= \frac{\mathcal{M}_0 \phi_1 \ell_1}{\alpha - x}\,\{1+G(t)\}\left(\frac{t}{\tau_1}\right)^{\frac{\alpha-x}{1-\alpha}} \qquad (8.16)\end{aligned}$$

Cluster Evolution and q_0

A traditional method of measuring q_0, which dates back (in idea) all the way to Hubble, is to use the brightest elliptical galaxies as a standard candle, and look for curvature in the Hubble diagram. Recall from the very first homework assignment that, when the equation for distance modulus in an Einstein-de Sitter universe is expanded out in a power series,

$$m_{\rm bol} = M_{\rm bol} + 5\log\left(\frac{cz}{H_0}\right) + 1.086(1 - \frac{\Omega_0}{2})\, z \tag{8.17}$$

So, as z increases, the relation between $m_{\rm bol}$ and $\log z$ increasingly departs from a straight line Hubble law, in a manner that depends on q_0 (or Ω_0 and Λ). In the $\Lambda = 0$ case, high-z galaxies will appear brighter than they should (according to the linear Hubble law) by

$$\Delta m = 1.086(1 - \frac{\Omega_0}{2})z \tag{8.18}$$

Now consider what passive stellar evolution will do to the Hubble diagram. Let's assume that all the stars in an elliptical galaxy were born (essentially) at the same time. From (8.13)

$$\ln \mathcal{L}_T = \ln \frac{\mathcal{M}_0 \phi_1 \ell_1}{\alpha - x} + \ln[1 + G(t)] - \frac{\alpha - x}{1 - \alpha}\ln \tau_1 + \frac{\alpha - x}{1 - \alpha}\ln t \tag{8.19}$$

so

$$\begin{aligned}\frac{d\ln \mathcal{L}_T}{d\ln t} &= \frac{\alpha - x}{1 - \alpha} + \frac{d}{dt}\left\{\ln[1 + G(t)]\right\}\frac{dt}{d\ln t} \\ &= \frac{\alpha - x}{1 - \alpha} + \frac{t}{1 + G(t)}\frac{dG}{dt}\end{aligned} \tag{8.20}$$

We can find the derivative of G with respect to t via (8.13). If we do that, then do a little mathematical manipulation, we arrive at

$$E = \frac{d\ln\mathcal{L}}{d\ln t} = \frac{1}{\alpha - 1}\left\{x - \alpha + \frac{G}{G+1}\right\} \tag{8.21}$$

With a Salpeter mass function, $\alpha \sim 3.5$, and $G = 6$, $E \approx -0.5$.

How bad is this? For an Einstein-de Sitter universe, the lookback time for this galaxy is

$$\begin{aligned}
\Delta t &= \frac{2}{3}\frac{1}{H_0}\left\{1 - (1+z)^{-3/2}\right\} \\
&\approx \frac{2}{3}\frac{1}{H_0}\left\{1 + \frac{3}{2}(1+z)^{-5/2}_{z=0} \cdot z + \ldots\right\} \\
&\approx \frac{z}{H_0}
\end{aligned} \tag{8.22}$$

where we have simply expanded the equation for lookback time in a power series near $z = 0$. Using this, the luminosity evolution that has occurred between redshift z and today is

$$\ln\mathcal{L}_z - \ln\mathcal{L}_0 = E\left\{\ln(t_0 - \frac{z}{H_0}) - \ln t_0\right\} \tag{8.23}$$

If we convert $\ln\mathcal{L}$ to magnitude, and again expand the resulting equation in a Taylor series about $z = 0$ (*i.e.*, using $\ln(1+x) \approx x$), then

$$\begin{aligned}
\Delta m &= -2.5\log e \cdot E\ln\left\{1 - \frac{z}{H_0 t_0}\right\} \\
&\approx 1.086E\left(\frac{z}{H_0 t_0}\right) \approx 1.086Ez
\end{aligned} \tag{8.24}$$

The above equation says that at large z, galaxies will appear much brighter than they should according to the Hubble law. Moreover, if you set (8.18) and (8.24) equal

$$\Delta m = 1.086(1 - \frac{\Omega_0}{2} z) = 1.086 E z \Longrightarrow \frac{\Omega_0}{2} = 1 - E \approx 1.5 \quad (8.25)$$

In other words, if you want to use elliptical galaxies as standard candles, you must correct for the passive evolution of their stars, and the amount of this correction is large!

$$\Delta\Omega_0 = \Omega_0(\text{observed}) - \Omega_0(\text{true}) = 3 \quad (8.26)$$

Photometric Mass-to-Light Ratio

In addition to a population's total luminosity, there are several other quantities which can be computed from first principles. First is the photometric mass-to-light ratio. Recall the total luminosity of main sequence stars in a stellar population is

$$\mathcal{L}_D = \frac{M_0 \phi_1 \ell_1}{\alpha - x} \left\{ m_{tn}^{\alpha - x} - m_L^{\alpha - x} \right\} \tag{8.06}$$

where m_{tn} is the turnoff mass of the main sequence. As stated previously, $\alpha - x$ is usually greater than zero, so the last term of this equation is negligible; stars at the low end of the mass sequence do not contribute significantly to the system's luminosity.

On the other hand, low mass stars can be an important part of the cluster's mass. From (8.02), the visible mass of stars in a galaxy is given by

$$\begin{aligned} \mathcal{M} &= M_0 \int_{m_L}^{m_U} \phi(m) m \, dm = M_0 \int_{m_L}^{m_U} \phi_1 m^{-x} dm \\ &= \frac{M_0 \phi_1}{1 - x} \left\{ m_{tn}^{1-x} - m_L^{1-x} \right\} \end{aligned} \tag{8.27}$$

(neglecting the mass in invisible stellar remnants). If $x > 1$ (as is normally assumed), the exponents in (8.27) are less than zero, and the value of $\mathcal{M}$ is dominated by the last term in the equation. Physically, this means that most of the stellar mass of a population resides in low mass stars.

The combination of (8.06) and (8.27) means that it is virtually impossible to determine a population's photometric mass-to-light ratio from observations alone. One can always drive up $\mathcal{M}/\mathcal{L}$ by postulating the existence of low-mass stars which add mass (via 8.27), without adding luminosity (8.06).

Stellar Evolutionary Flux

An interesting property associated with stellar populations is the stellar evolutionary flux, *i.e.*, the number of stars passing through any (post main-sequence) phase of evolution at a given time. Just as the amount of water flowing down a river is controlled by the spillway of a dam, the number of stars evolving through various stages of evolution is controlled by the rate at which these stars move off the main sequence. (All other rates are much faster than this.) As we have seen previously, this rate is

$$N_{tn} = M_0 \phi(m_{tn}) \frac{dm_{tn}}{dt} = M_0 \phi_1 m^{-(1+x)} \frac{dm_{tn}}{dt} \tag{8.28}$$

From (8.05)

$$m = \left(\frac{t}{\tau_1}\right)^{1/1-\alpha} \implies \frac{dm_{tn}}{dt} = \frac{1}{\tau_1(1-\alpha)} \left(\frac{t}{\tau_1}\right)^{\alpha/1-\alpha} \tag{8.29}$$

so the stellar evolutionary flux is

$$N_{tn} = \frac{M_0 \phi_1}{\tau_1(1-\alpha)} \left(\frac{t}{\tau_1}\right)^{\frac{x-1-\alpha}{\alpha-1}} \tag{8.30}$$

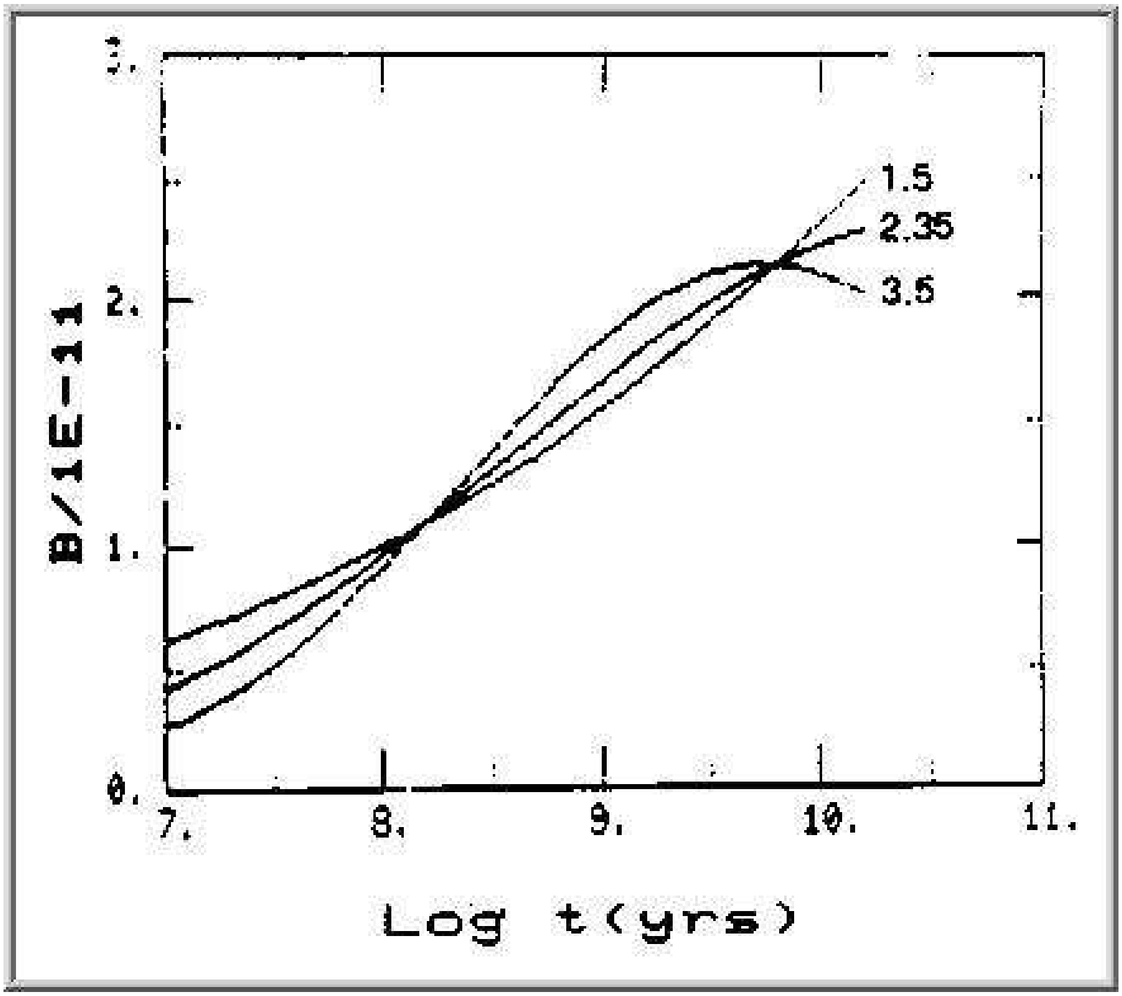
Naturally, the number of stars evolving through any phase of evolution in a galaxy is proposal to the number of stars in the galaxy. The best way to take this dependence out is to normalize the evolutionary flux to the size of the galaxy. This cannot be done with mass, since we do not know the total number of stars that are present in the system. However, we do know the population's luminosity, which is defined in (8.16). So, when we normalize (8.30) to this luminosity, we arrive at the population's **luminosity-specific stellar evolutionary flux**. After a bit of math, this quantity comes out to be

$$B = \frac{N_{tn}}{L_t} = \frac{\alpha - x}{(\alpha-1)(1+G(t))\ell_1 \tau_1} \left(\frac{t}{\tau_1}\right)^{\frac{1}{\alpha-1}} \tag{8.31}$$

Note that the exponent of time is less than 1; thus B is rather insensitive to the age of the stellar population. (The difference between the stellar evolutionary flux of a 7 Gyr stellar population and a 12 Gyr population is less than a 25%.) Similarly, the stellar evolutionary flux is not very sensitive to the exact value of x.

Detailed calculations show that the luminosity-specific stellar evolutionary flux is $B \approx 2 \times 10^{-11}$ stars yr^{-1} $L_{\odot}^{-1}$ for all old (or moderately old) stellar populations. This makes is easy to predict the number of any post-main sequence type of star present in a galaxy: since you know B, and you can measure the luminosity of the galaxy, all you need is the lifetime of the phase in question. For example, the lifetime of a planetary nebula is $\tau \sim 25,000$ years. If a galaxy is observed to have an absolute luminosity of $\mathcal{L} = 10^{11}\mathcal{L}_{\odot}$, then the number of PN in the galaxy is $N(PN) \approx B \cdot \mathcal{L} \cdot \tau = 50,000$.

Note: a common idea these days is that all (or many) planetary nebulae are formed out of binary star interactions. If so, then the previous calculation may not work, since it does not take into account binary-star interactions. (Interestingly, however, the estimate does appear to come close to predicting PN numbers.) Similarly, if not all stars evolve through a given stage, then the calculation may be in error.



The plot above shows how a population's bolometric-luminosity specific stellar evolutionary flux changes with the systems age and IMF. Note that the x-axis (population age) is a log quantity, while the y-axis is linear. All old (> 3 Gyr) stellar population have a rate of $\sim 2 \times 10^{11}$ stars per year per solar luminosity, independent of age, IMF, or metallicity.

[Renzini & Buzzoni 1986, in *Spectral Evolution of Galaxies*, p. 195]

Mass Loss from Stars

Another useful quantity to know is the rate of mass loss from stars as a function of time. As before, the key to calculating this is to realize that almost all the mass lost from stars comes during the post main-sequence phase of evolution. Thus, the rate at which a population of stars loses mass is just by the rate at which stars move off the main sequence times the amount of mass each star loses. If m is the initial mass of the star, and w the mass of the stellar remnant (*i.e.*, white dwarf), then the mass ejection rate is

$$E(t) = N_{tn}(m_{tn} - w) = M_0 \phi(m_{tn}) \frac{dm_{tn}}{dt}(m_{tn} - w) \qquad (8.32)$$

After performing the same substitutions as was done for the calculation of stellar evolutionary flux, this simplifies to

$$E(t) = \frac{M_0 \phi_1}{\tau_1 (1 - \alpha)}(m_{tn} - w) \left(\frac{t}{\tau_1} \right)^{\frac{x-1-\alpha}{\alpha-1}} \qquad (8.33)$$

Once again, if we normalize to luminosity via (8.16), we can derive the luminosity specific mass loss rate

$$\frac{E(t)}{\mathcal{L}_t} = \frac{\alpha - x}{1 + G(t)} \frac{(m_{tn} - w)}{\alpha - 1} \frac{1}{\ell_{tn} t} \qquad (8.34)$$

Numerically, this works out to a mass loss rate of $\sim 0.02\mathcal{M}_\odot$ per Gigayear per unit solar luminosity for an old (10^{10} yr) stellar population. When integrated over the lifetime of a galaxy, the result is that $\sim 15\%$ of the initial stellar mass will be lost during a Hubble time.

More Complex Systems

Real galaxies usually contain multiple stellar populations, each with its own mass, age, and metallicity. Computing the behavior of such a system simply means summing variations SSP components. It would, however, be useful to some zeroth order approximation to know the mix of stellar populations within a galaxy.

A common way to do this is to simply assume that the star-formation rate of a galaxy declines exponentially with time, with some decay rate τ, *i.e.*,

$$\mathcal{M}(t) \propto e^{-t/\tau} \tag{8.35}$$

Irregular and late-type spiral galaxies are assigned large values of τ, suggesting an almost constant star-formation rate over the history of the universe. Elliptical galaxies have small values of τ, thus reflecting the fact that their star-formation has e-folded away long ago. Note however, that, many/most galaxies likely undergo a succession of discrete starbursts, with low-level star formation occuring in between times of extreme activity. So these models may be of only moderate use.

Problems in Photometric Evolution Calculations

The above analytic solutions are only useful for guidance. To compare with observations, predictions must be made in specific bandpasses, or for specific absorption lines. This requires numerical calculations which include

1) sets of stellar isochrones, detailing the precise number of stars at any position in the HR diagram as a function of age.
2) sets of model stellar atmospheres, giving the emitted spectrum (or broadband color) of each star, as a function of its temperature, surface gravity, and metallicity.
3) an understanding of various non-stellar processes that may effect the emergent spectrum from galaxies. Such effects include internal extinction due to dust, the presence of emission-lines from ionized gas, and (for high-redshift objects) absorption due to the intervening intergalactic medium (*i.e.,* the Lyα forest).

Today, the most controversial parts of these codes include their treatment of thermally pulsing AGB stars, and the proper modelling of the horizontal branch and post-AGB stars. The former is important since it involves extremely luminous stars, which, due to their mass loss, may be obscured by circumstellar extinction. The latter is important for modeling old stellar systems which have very few blue stars. For these objects, a small change in the morphology of the horizontal branch (*i.e.,* the ratio of red to blue objects), or the number of slow-evolving post-AGB stars can change the system's UV color dramatically.